EE 215 Fundamentals of Electrical Engineering — Spring 2010

Problem #6 Solution

P 5.12 [a] This circuit is an example of an inverting summing amplifier.

[b] $v_o = -\frac{220}{44}v_a - \frac{220}{27.5}v_b - \frac{220}{80}v_c = -5 - 12 + 11 = -6\ \text{V}$

[c] $v_o = -6 - 8v_b = \pm 10$

$$\therefore\quad v_b = -0.5\ \text{V} \quad \text{when} \quad v_o = 10\ \text{V};$$

$$v_b = 2\ \text{V} \quad \text{when} \quad v_o = -10\ \text{V}$$

$$\therefore\quad -0.5\ \text{V} \le v_b \le 2\ \text{V}$$

P 5.18 [a] $v_p = v_n = \frac{68}{80}v_g = 0.85v_g$

$$\therefore\quad \frac{0.85v_g}{30{,}000} + \frac{0.85v_g - v_o}{63{,}000} = 0;$$

$$\therefore\quad v_o = 2.635v_g = 2.635(4), \quad v_o = 10.54\ \text{V}$$

[b] $v_o = 2.635v_g = \pm 12$

$$v_g = \pm 4.55\ \text{V}, \qquad -4.55 \le v_g \le 4.55\ \text{V}$$

[c] $\frac{0.85v_g}{30{,}000} + \frac{0.85v_g - v_o}{R_f} = 0$

$$\left(\frac{0.85R_f}{30{,}000} + 0.85\right)v_g = v_o = \pm 12$$

$$\therefore\quad 1.7R_f + 51 = \pm 360; \qquad 1.7R_f = 360 - 51; \qquad R_f = 181.76\,\text{k}\Omega$$

P 5.26 Use voltage division to find v_p:

$$v_p = \frac{2000}{2000 + 8000}(5) = 1\ \text{V}$$

Write a KCL equation at v_n and solve it for v_o:

$$\frac{v_n - v_a}{5000} + \frac{v_n - v_o}{R_f} = 0 \qquad \text{so} \qquad \left(\frac{R_f}{5000} + 1\right)v_n - \frac{R_f}{5000}v_a = v_o$$

Since the op amp is ideal, $v_n = v_p = 1$V, so

$$v_o = \left(\frac{R_f}{5000} + 1\right) - \frac{R_f}{5000} v_a$$

To satisfy the equation,

$$\left(\frac{R_f}{5000} + 1\right) = 5 \qquad \text{and} \qquad \frac{R_f}{5000} = 4$$

Thus, $R_f = 20$ kΩ.

P 5.31 $v_p = \frac{1500}{9000}(-18) = -3 \text{ V} = v_n$

$$\frac{-3 + 18}{1600} + \frac{-3 - v_o}{R_\text{f}} = 0$$

$$\therefore \quad v_o = 0.009375 R_\text{f} - 3$$

$$v_o = 9 \text{ V}; \qquad R_\text{f} = 1280\,\Omega$$

$$v_o = -9 \text{ V}; \qquad R_\text{f} = -640\,\Omega$$

But $\quad R_\text{f} \geq 0, \qquad \therefore \quad R_\text{f} = 1.28\,\text{k}\Omega$

P 6.1 $0 \leq t \leq 2\,\text{s}:$

$$i_L = \frac{10^3}{2.5}\int_0^t 3 \times 10^{-3} e^{-4x}\,dx + 1 = 1.2 \frac{e^{-4x}}{-4}\Bigg|_0^t + 1$$

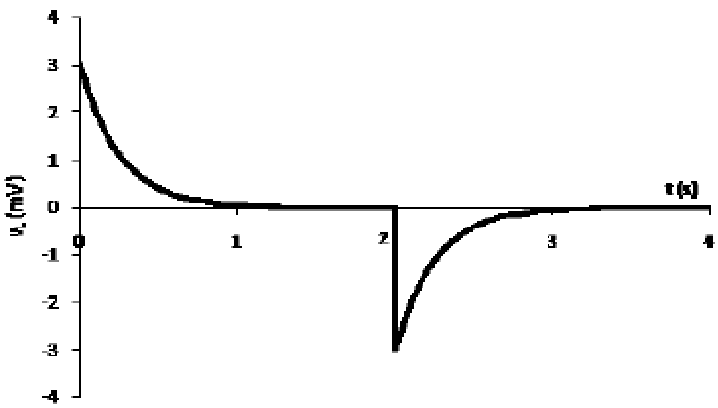
$$= -0.3e^{-4t} + 1.3\,\text{A}, \qquad 0 \leq t \leq 2\,\text{s}$$

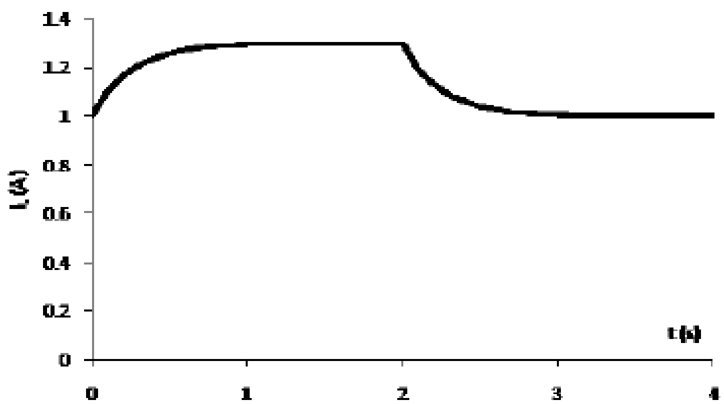
$$i_L(2) = -0.3e^{-8} + 1.3 = 1.3\,\text{A}$$

$t \geq 2\,\mathrm{s}:$

$$i_L = \frac{10^3}{2.5}\int_2^t -3\times 10^{-3}e^{-4(x-2)}\,dx + 1.3 = -1.2\frac{e^{-4(x-2)}}{-4}\Big|_2^t + 1.3$$

$$= 0.3e^{-4(t-2)} + 1\,\mathrm{A}, \qquad t \geq 2\,\mathrm{s}$$





P 6.6 [a] $i(0) = A_1 + A_2 = 0.04$

$$\frac{di}{dt} = -10{,}000A_1e^{-10{,}000t} - 40{,}000A_2e^{-40{,}000t}$$

$$v = -200A_1e^{-10{,}000t} - 800A_2e^{-40{,}000t}\,\mathrm{V}$$

$$v(0) = -200A_1 - 800A_2 = 28$$

Solving, $A_1 = 0.1$ and $A_2 = -0.06$

Thus,

$$i_1 = (100e^{-10,000t} - 60e^{-40,000t})\,\text{mA} \qquad t \geq 0$$

$$v = -20e^{-10,000t} + 48e^{-40,000t}\,\text{V}, \qquad t \geq 0$$

[b] $i = 0 \quad \text{when} \quad 100e^{-10,000t} = 60e^{-40,000t}$

Therefore

$$e^{30,000t} = 0.6 \quad \text{so} \quad t = -17.03\,\mu\text{s} \quad \text{which is not possible!}$$

$$v = 0 \quad \text{when} \quad 20e^{-10,000t} = 48e^{-40,000t}$$

Therefore

$$e^{30,000t} = 2.4 \quad \text{so} \quad t = 29.18\,\mu\text{s}$$

Thus the power is zero at $t = 29.18\,\mu$s.

P 6.21 $5\|(12 + 8) = 4\,\text{H}$

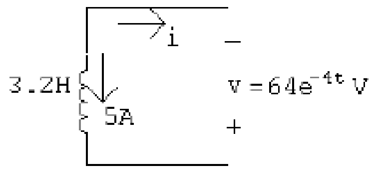
$4\|4 = 2\,\text{H}$

$15\|(8 + 2) = 6\,\text{H}$

$3\|6 = 2\,\text{H}$

$6 + 2 = 8\,\text{H}$

P 6.25 **[a]**



$$3.2\frac{di}{dt} = 64e^{-4t} \qquad \text{so} \qquad \frac{di}{dt} = 20e^{-4t}$$

$$\begin{aligned} i(t) &= 20\int_0^t e^{-4x}\,dx - 5 \\ &= 20\frac{e^{-4x}}{-4}\Big|_0^t - 5 \\ i(t) &= -5e^{-4t}\,\text{A} \end{aligned}$$

[b] $4\frac{di_1}{dt} = 64e^{-4t}$

$$\begin{aligned} i_1(t) &= 16\int_0^t e^{-4x}\,dx - 10 \\ &= 16\frac{e^{-4x}}{-4}\Big|_0^t - 10 \\ i_1(t) &= -4e^{-4t} - 6\,\text{A} \end{aligned}$$

[c] $16\frac{di_2}{dt} = 64e^{-4t} \qquad \text{so} \qquad \frac{di_2}{dt} = 4e^{-4t}$

$$\begin{aligned} i_2(t) &= 4\int_0^t e^{-4x}\,dx + 5 \\ &= 4\frac{e^{-4x}}{-4}\Big|_0^t + 5 \\ i_2(t) &= -e^{-4t} + 6\,\text{A} \end{aligned}$$

[d] $p = -vi = (-64e^{-4t})(-5e^{-4t}) = 320e^{-8t}\,\text{W}$

$$\begin{aligned} w &= \int_0^\infty p\,dt = \int_0^\infty 320e^{-8t}\,dt \\ &= 320\frac{e^{-8t}}{-8}\Big|_0^\infty \\ &= 40\,\text{J} \end{aligned}$$

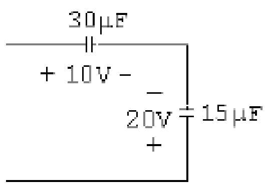
[e] $w = \frac{1}{2}(4)(-10)^2 + \frac{1}{2}(16)(5)^2 = 400\,\text{J}$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 400 - 40 = 360\,\text{J}$

[g] $w_{\text{trapped}} = \frac{1}{2}(4)(-6)^2 + \frac{1}{2}(16)(6)^2 = 360\,\text{J}$ checks

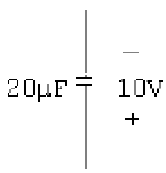
P 6.26 $\frac{1}{C_1} = \frac{1}{48} + \frac{1}{16} = \frac{1}{12}; \qquad C_1 = 12\,\mu\text{F}$

$C_2 = 3 + 12 = 15\,\mu\text{F}$

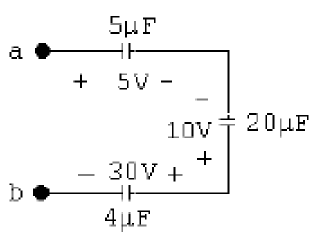


$\frac{1}{C_3} = \frac{1}{30} + \frac{1}{15} = \frac{1}{10}; \qquad C_3 = 10\,\mu\text{F}$

$C_4 = 10 + 10 = 20\,\mu\text{F}$



$\frac{1}{C_5} = \frac{1}{5} + \frac{1}{20} + \frac{1}{4} = \frac{1}{2}; \qquad C_5 = 2\,\mu\text{F}$



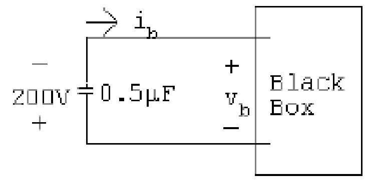
Equivalent capacitance is $2\,\mu\text{F}$ with an initial voltage drop of +25 V.

P 6.29 $\frac{1}{C_e} = \frac{1}{1} + \frac{1}{5} + \frac{1}{1.25} = \frac{10}{5} = 2$

$\therefore \quad C_2 = 0.5\,\mu\text{F}$

$v_b = 20 - 250 + 30 = -200\,\text{V}$

[a]



$$v_{\rm b} = -\frac{10^6}{0.5}\int_0^t -5\times10^{-3}e^{-50x}\,dx - 200$$

$$= 10{,}000\frac{e^{-50x}}{-50}\Big|_0^t - 200$$

$$= -200e^{-50t}\,\text{V}$$

[b] $$v_{\rm a} = \frac{10^6}{5}\int_0^t -5\times10^{-3}e^{-50x}\,dx - 20$$

$$= 20(e^{-50t}-1)-20$$

$$= 20e^{-50t}-40\,\text{V}$$

[c] $$v_{\rm c} = \frac{10^6}{1.25}\int_0^t -5\times10^{-3}e^{-50x}\,dx - 30$$

$$= 80(e^{-50t}-1)-30$$

$$= 80e^{-50t}-110\,\text{V}$$

[d] $$v_{\rm d} = 10^6\int_0^t -5\times10^{-3}e^{-50x}\,dx + 250$$

$$= 100(e^{-50t}-1)+250$$

$$= 100e^{-50t}+150\,\text{V}$$

CHECK: $$v_{\rm b} = -v_{\rm c} - v_{\rm d} - v_{\rm a}$$

$$= -200e^{-50t}\,\text{V} \quad \text{(checks)}$$

[e] $$i_1 = 0.2\times10^{-6}\frac{d}{dt}[100e^{-50t}+150]$$

$$= 0.2\times10^{-6}(-5000e^{-50t})$$

$$= -e^{-50t}\,\text{mA}$$

[f] $$i_2 = 0.8\times10^{-6}\frac{d}{dt}[100e^{-50t}+150]$$

$$= -4e^{-50t}\,\text{mA}$$